The Embedding Transform.
A Novel Analysis of Non-Stationarity in the EEG

Carlos A. Loza and Jose C. Principe

***Abstract*— We introduce a novel technique to analyze non-stationarity in single-channel Electroencephalogram (EEG) traces: the Embedding Transform. The approach is based on Walter J. Freeman's studies concerning active and rest stages and deviations from Gaussianity. Specifically, we generalize his idea in order to include cases where the neuromodulations are sparse in time. Specifically, the transform maps the temporal sequences to a set of ℓ^2-norms where modulated patters are emphasized. In this way, the background, chaotic activity can be modeled as the main lobe of the distribution, while the relevant synchronizations (or desynchronizations) fall on the right (or left) tail of the density of such norms. We test the algorithm on two different datasets: alpha bursts on synthetic data simulated in the BESA software and low-gamma oscillations in the motor cortex from the Brain-Computer Interface (BCI) Competition 4 Dataset 4. The results are promising and place the Embedding Transform as a quick, single-parameter tool to effectively assess which channels (or regions) are actively engaged in particular behaviors and which are in a more silent type of stage.**

***Index Terms*— EEG, Embedding, Gaussianity, Non-Stationarity,**

I. Introduction

Extracellular electrical potentials from the brain such as Electroencephalogram (EEG), Electrocorticogram (ECoG), and Local Field Potentials (LFP) share the common property of spatio-temporal synchronization epochs expressed as neuromodulations or phasic events over time [1]. These well-defined temporal signatures constitute the collective effort of neuronal assemblies in order to process external stimuli and/or regulate internal physiological phenomena. Proper and systematic analysis of such events is reflected on the myriad of Neuroengineering studies where novel properties and network dynamics from the brain are uncovered time after time, e.g. sleep stages [2], epilepsy studies [3], Brain-Machine Interfaces [4], topographical and functional mapping [5], the physiology of movement disorders [6], and memory encoding [7] to name a few.

Most of these studies are, sometimes inadvertently, anchored on the key concept of perpetual transitions in the brain: from highly complex unpredictable chaos to robust, transiently predictable (and oscillatory) stages. In dynamical systems jargon, this transitional state is known as criticality—a type of stability where the network reorganizes its dynamics to and fro due to external and/or internal perturbations [8]. This is translated as non-stationarity in the temporal traces of the EEG, e.g. the network dynamics (neuromodulations for instance) vary significantly from rest to active stages, or even within a single active setting. The challenge is to properly exploit statistical and Machine Learning techniques in an environment where non-stationarity is the norm in order to provide quantifiable conclusions.

The simplest solution involves strict experimental control, e.g. 2-class supervised classification problems with fixed and controlled epoch duration [9]. Another alternative involves subspace analysis [10], which might be effective; however, further interpretation becomes unclear due to the inherent mixing of the sources. Lastly, temporal segmentation is favored from a Neurophysiological point of view, i.e. the signal can be regarded as quasi-stationary during the temporal window under analysis; nevertheless, the optimal segmentation interval is still subject to debate [11] and, hence, is an application-dependent parameter [12], [2].

Our proposed solution goes along the lines of previous studies that focused on stationarity as well as on the specifics regarding EEG dynamics [13], [14], [15]. Walter J. Freeman was a pioneer in the Neurophysiology and Computational Neuroscience fields; in particular, he posited the theory of linking statistical concepts to behavioral stages via higher-order moments and deviations from Gaussianity [16]. We base our work on such ideas and generalize them in order to characterize relevant neuromodulations and background activity in a different domain where time is explicitly taken into account. The result is the so-called Embedding Transform; this novel technique allows to map bandpassed EEG traces to a set of ℓ^2-norms where it is possible to determine which recordings are actively engaged and which ones are in a silent, background type of stage. We test the proposed algorithm on synthetic data and BCI traces with promising results that place the Embedding Transform as a quick, intuitive, single-parameter tool to isolate relevant areas, channels, and traces in order to improve further processing and interpretation. The rest of the paper is organized as follows: Section 2 introduces the Embedding Transform alongside its main properties and implementation. Section 3 describes the experimental results, and lastly, Section 4 concludes the paper and discusses further work.

II. The Embedding Transform

Freeman [16] showed experimentally that the probability density function (pdf) of the ECoG traces from the cat's olfactory bulb changes according to the animal's behavioral stage. For rest periods, the ECoG amplitudes would resemble a Gaussian distribution, while for active stages,

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the densities would deviate from Gaussianity according to estimated higher-order moments, such as kurtosis. The result is a model that links stationarity in the time domain to Gaussianity of the amplitudes. With this idea in mind, we generalize Freeman's hypothesis in order to emphasize the neuromodulations which are normally the main scope of quantitative analysis.

Let $y[n]$ be a single-channel, bandpassed EEG trace corresponding to a particular behavioral state under study. We introduce a novel transformation that explicitly models the temporal neuromodulations by means of embedding the sequence in a M-dimensional space with constraints that emphasize modulated patterns. Specifically, $\{\beta_M(\tilde{y}[n])_k\}_k$ is a set of ℓ^2-norms from the M-sample long snippets (M-snippets) extracted from $y[n]$:

$$\begin{aligned}\{\beta_M(y[n])_k\}_k =& ||y[\pi_k - M/2 : \pi_k + M/2]||_2 \\ & \text{s.t.} \quad \pi \in \Pi, \quad k = 1, \ldots, |\Pi|\end{aligned} \tag{1}$$

where Π is the set of middle-point indices corresponding to all the potential non-overlapping M-snippets in $y[n]$. Moreover, instead of being a mere ℓ^2-norm mapping in a M-dimensional embedding, $\{\beta_M(y[n])_k\}_k$, or simply $\beta_M(y[n])$, is built in a sequential manner: it starts with the M-snippets that display clear modulatory, spindle-shaped patterns and, then, continues with the remaining M-snippets from $y[n]$. In this way, the relevant temporal information (embedded in the neuromodulations) is not only preserved, but emphasized by means of the priority given to such patterns. The process can be extended to multi-trial, single channel, bandpassed EEG recordings corresponding to the phenomenon under consideration. Algorithm 1 details the implementation for the Embedding Transform in the case of N_i-sample long P trials.

The functions *envelope*, *smooth*, and *peaks* compute the Hilbert Transform magnitude of the temporal sequence, smooth it ($M/2$-span moving average), and estimate its relevant peaks, respectively. The function *findMsnippets* searches for potential whole M-snippets remaining in the residue. The algorithm terminates when all possible M-snippets have been retrieved. The result is a set of ℓ^2-norms where the relevant neuromodulations are extracted first and well-isolated from the unmodulated, chaotic, background type of activity.

Some of the properties of the Embedding Transform include non-negativity, lower and upper bounds on its cardinality, and, for completeness, special cases for specific values of its main parameter, M:

$$\beta_1(y[n]) = |y[n]| \tag{2}$$

$$\beta_N(y[n]) = \sigma_y \tag{3}$$

where equation 2 depicts the special case where Algorithm 1 would not find relevant peaks in the smoothed Hilbert Transform magnitude; thus, the Embedding Transform defaults to the EEG ℓ^1-norm. For $\beta_2(y[n])$, the mapping simply downsamples (by 2) the sequence $|y[n]|$. Additionally, when $N = M$, i.e. the only M-snippet is the entire EEG sequence itself, by applying equation 1, the result is the biased estimator of the standard deviation of $y[n]$, σ_y.

Algorithm 1 Embedding Transform.

Input: $\{y_i[n]\}_{i=1}^P, M$
Output: $\{\beta_M(y[n])_k\}_k$

```
k ← 1
for i = 1, ..., P do
    r_i[n] ← 1_{N_i × 1}
    y'_i[n] = envelope(y_i[n])
    s_i[n] = smooth(y'_i[n], M)
    {ζ_j}_{j=1}^p = peaks(s_i[n], M)
    for j = 1, ..., p do
        β_M(y[n])_k ← ||y_i[ζ_j − M/2 : ζ_j + M/2]||_2
        r_i[ζ_j − M/2 : ζ_j + M/2] = 0_{M × 1}
        k ← k + 1
    end for
    CONTINUE = TRUE
    while CONTINUE == TRUE do
        ζ ← findMsnippets(r_i[n], M)
        if ζ ≠ ∅ then
            β_M(y[n])_k ← ||y_i[ζ − M/2 : ζ + M/2]||_2
            r_i[ζ − M/2 : ζ + M/2] = 0_{M × 1}
            k ← k + 1
        else
            CONTINUE = FALSE
        end if
    end while
end for
```

The rationale behind the Embedding Transform comes mainly from one of its key properties: if the amplitudes of the bandpassed trace $y[n]$ conform to a zero-mean Gaussian distribution, then $\beta_M(y[n])$ can be modeled as a chi distribution with M degrees of freedom. Furthermore, for large M (usually satisfied with high sampling rates), the chi distribution reduces to a Gaussian one by the Central Limit Theorem [17]. Hence, Freeman's hypothesis is preserved for bandpassed background activity without neuromodulations. On the other hand, if $y[n]$ does not conform to the Gaussianity assumption, the distribution of $\beta_M(y[n])$ will be modified accordingly, e.g. if the network is driven by excitatory, synchronization-type of inputs, the resulting pdf will be right-tailed (positive skewness); on the contrary, if the active network is of the inhibitory, desynchronization-type, then the pdf will be left-tailed (negative skewness).

Strictly speaking, the Embedding Transform has two main hyperparameters: the bandpass filter coefficients and the length of the potential M-snippets (M in samples). The first one can be designed depending on the rhythm under study, while the second one should be selected according to Neurophysiological principles, previous studies, visual inspection by experts, or quantitative tools, e.g. Time-Frequency (TF) Transform. When properly estimated, the M parameter will derive in a mapping that compromises between strict time-based analysis which disregards modulated power in the EEG

($M = 1$) and power-based techniques that ignore the inherent temporal variation of averaged neuronal potentials ($M = N$).

III. Experimental Results

The proposed hypothesis was tested utilizing a set of synthetic EEG from a software package and a BCI Competition ECoG dataset. We mainly focused on the Embedding Transform as an exploratory tool that can potentially isolate relevant "active" electrodes via analysis of the higher-order moments of the resulting $\beta_M(y[n])$ pdf. It is worth noting that the Embedding Transform was briefly introduced in [18] and later implemented as part of the noise model in a Marked Point Process framework for the EEG [19]; however, this is the first contribution where the main focus is the Embedding Transform and its potential as a stand-alone technique to analyze the stationarity in extracellular electrical potentials.

A. Synthetic Data

The set of single-channel, multi-trial traces was simulated using the BESA software [20]. In particular, we focused on a single source (Cz) according to the EEG 10-20 system and a default spherical head model that takes into account propagation delays and amplitude attenuations. The duration of the alpha neuromodulations was set to 1 second (sampling frequency of 100 Hz) with a maximum peak-to-peak amplitude of $5\,\mu V$. The trial duration was set in an interval from 30 to 45 s. The number of phasic events was controlled in order to simulate diverse scenarios, while the mean interval and interval variation between adjacent neuromodulations were varied as well to emulate real conditions. Lastly, correlated pink noise was added to the traces with different RMS levels.

For this case, the M parameter is well known from the ground truth, i.e. $M = 100$. The alpha band (8 - 12 Hz) was isolated from the simulated EEG using a 5th-order Butterworth filter with quality factor $Q = 1$. Fig. 1 illustrates the pdf of the Embedding Transform for phasic event rates from 0 to 0.118 and a RMS level of $1.5\,\mu V$. The case of 0 neuromodulations (background noise only), corresponds to the electrode Fp1 under the assumption of a single source at Cz, i.e. the spatial attenuation derives in traces where no neuromodulations can be distinguished from the noise activity. It is evident that, as the phasic events become more prevalent, the distribution starts to shift from a Gaussian-like shape to a right-skewed density and, finally, to a nearly bimodal pdf. In general, the unmodulated activity (background) is modeled as the main lobe of the distribution, while on the other hand, the relevant modulated M-snippets are mapped to the right tail of the density. This confirms our hypothesis regarding rest and active stages; specifically, the neuromodulation rate dictates the shape of the distribution.

To quantify such change, the skewness of the $\beta_M(y[n])$ pdf is estimated for each noise and neuromodulation scenario. The results are summarized in Table I. As expected, the skewness for the 0 neuromodulation rate cases is very close to zero, i.e. resembling a Gaussian distribution. Then, there is a non-linear type of behavior when phasic events start to appear (high skewness); this is followed by decreases

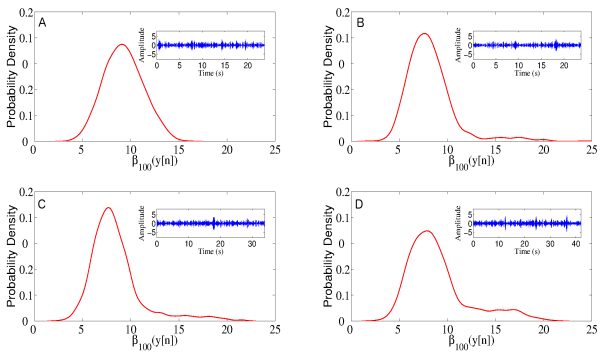

Fig. 1. Probability density function of the Embedding Transform for simulated synthetic data using the BESA software for different neuromodulation rates (A: 0, B: 0.038, C: 0.061, D: 0.118 phasic events per second). Alpha band. Insets show bandpassed sample traces for each case.

TABLE I
Skewness of the Embedding Transform distribution for different simulated cases. BESA-generated synthetic EEG.

RMS (μV)	Rate (Neuromodulations per second)			
	0	0.038	0.061	0.118
1.0	0.116	2.822	2.445	1.794
1.5	0.164	1.951	1.852	1.312
2.0	0.247	1.052	0.992	1.045

in the skewness which seem to be heavily dependent on the noise level. In the limit, the estimated skewness should decrease to a value close to zero when the phasic events completely outnumber the unmodulated M-snippets; also in this case, the overall pdf mean would shift from the noise power (rate equal to 0) to the power of traces with full, dense neuromodulation patterns (rate equal to 1).

B. BCI Competition Data

The second set of results utilizes the BCI Competition IV Dataset 4 [21]. The main goal of the competition was to infer the flexion of individual fingers from multi-channel ECoG traces recorded subdurally from the motor cortex (contralateral to the hand under study). In addition, a data glove recorded the cued finger flexions. The subjects were asked to move a particular finger depending on the visual cue (2-second long). After each cue, a 2-second long rest period followed, and, during each finger flexion task, the subjects typically performed 3—5 consecutive movements or taps. Therefore, timestamps of visual stimulus presentation, finger flexion kinematics, and digitized (sampling frequency 1000 Hz), bandpassed (0.15–200 Hz) ECoG traces were provided from a total of 3 subjects. For our analysis, the ECoG recordings were downsampled to 500 Hz.

We focused on the low-gamma band (76—100 Hz, 11-th order Butterworth filter, $Q = 2$) due to previous studies that link such rhythm to relevant amplitude and rate encoding mechanisms [21], [19]. M was estimated via visual inspection and TF analysis equal to 50 samples, i.e. 100 ms. After careful examination of the kinematics, we isolated the 2-

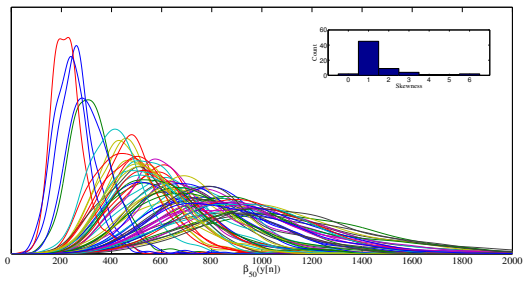

Fig. 2. Example of Embedding Transform distributions for Subject 3, finger 4 (BCI Competition IV Dataset 4). Low-γ band. 64 channels total. Inset shows the histogram of the estimated skewness for all 64 electrodes.

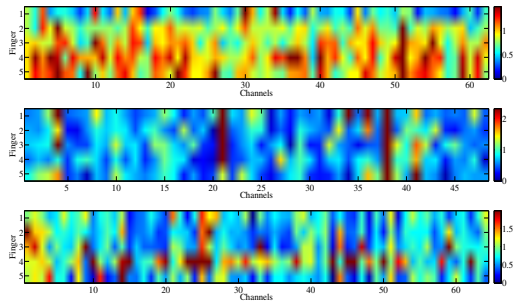

Fig. 3. Skewness of the Embedding Transform for all channels, finger tasks, and subjects of BCI Competition IV Dataset 4. Low-γ band (Color palette was truncated at the 95 percentile of skewness values per subject)

second excerpts where active finger flexion was observed; these constitute the trials in our framework. Fig. 2 depicts the densities after performing the Embedding Transform algorithm on a channel-by-channel scheme for a particular Subject/finger case. It is clear that the distributions are either symmetrical or right-skewed (see histogram inset), which reinforces the notion that these phasic events are of the excitatory, synchronization-driven type of patterns.

Moreover, Fig. 3 summarizes the results of all possible scenarios by estimating the skewness of the Embedding Transform pdf for each multi-trial, single-channel case. As expected, the values are non-negative and relatively sparse. This suggests that only a few channels are actively engaged in the motor activity, while others remain at a "silent" state where stationarity is expressed as low skewness values. This analysis can easily isolate the channels worth of further processing, and by doing so, reduce the computational load for such a high sampling rate, multi-channel type of experiment.

IV. Conclusions

We have formally introduced a novel technique to analyze the non-stationarity of bandpassed single-channel, multi-trial EEG traces: the Embedding Transform. This single-parameter algorithm is capable of isolating relevant actively engaged electrodes for further processing and determining the expected type of encoding (excitatory, synchronization-driven or inhibitory, desynchronization-driven). We demonstrated the potential of the proposed technique with two different datasets and rhythms. In the future, we expect to derive empirical rules to estimate neuromodulation rates based on the Embedding Transform alone. Lastly, the MATLAB code of the proposed method is available at `https://github.com/carlosloza/Embedding_Transform`.

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